

Write and prove a formula for $\sinh(x+y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.

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$$\begin{aligned} & \sinh x \cosh y + \cosh x \sinh y \quad (1) \\ &= \left[\frac{e^x - e^{-x}}{2} \frac{e^y + e^{-y}}{2} + \frac{e^x + e^{-x}}{2} \frac{e^y - e^{-y}}{2} \right] \quad (1/2) \\ &= \frac{(1) [e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}]}{4} + \frac{[e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y}]}{4} \quad (1) \\ &= \frac{(1/2) [2e^{x+y} - 2e^{-x-y}]}{(1/2) [4]} = \frac{[e^{x+y} - e^{-(x+y)}]}{2} = \sinh(x+y) \end{aligned}$$

Rewrite $\operatorname{sech}(\frac{1}{2} \ln 7)$ in terms of exponential functions and simplify.

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$$\boxed{\frac{2}{e^{\frac{1}{2} \ln 7} + e^{-\frac{1}{2} \ln 7}}} = \boxed{\frac{2}{\sqrt{7} + \frac{1}{\sqrt{7}}}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7+1} = \boxed{\frac{2\sqrt{7}}{8}} = \boxed{\frac{\sqrt{7}}{4}}$$

① ① ①/2 ①/2

Prove that $g(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$ is the inverse of $f(x) = \tanh x$ by simplifying $g(f(x))$.

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NOTE: You may use any identities proven in part [1] of the Hyperbolic Functions Supplement WITHOUT proving them here.

$$\boxed{\frac{1}{2} \ln \frac{1 + \tanh x}{1 - \tanh x}} \quad \textcircled{1}$$

$$\textcircled{1} = \boxed{\frac{1}{2} \ln \left(\frac{1 + \frac{e^{2x}-1}{e^{2x}+1}}{1 - \frac{e^{2x}-1}{e^{2x}+1}} \right)} \cdot \frac{e^{2x}+1}{e^{2x}+1}$$

$$\textcircled{1} = \boxed{\frac{1}{2} \ln \frac{e^{2x}+1+e^{2x}-1}{e^{2x}+1-(e^{2x}-1)}}$$

$$\textcircled{1} = \boxed{\frac{1}{2} \ln \frac{2e^{2x}}{2}} = \boxed{\frac{1}{2} \ln e^{2x}} \overset{\textcircled{\frac{1}{2}}}{=} \frac{1}{2} \cdot 2x = \boxed{x} \quad \textcircled{\frac{1}{2}}$$

SEE ALTERNATE

SOLUTIONS ON

OTHER VERSIONS

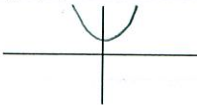
Sketch the general shape and position (including asymptotes) of the following graphs.
(Don't worry about specific x - or y - coordinates.)

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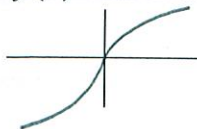
$$f(x) = \tanh x$$



$$f(x) = \cosh x$$



$$f(x) = \sinh^{-1} x$$



GRADED

BY ME

There is an identity involving $\sinh x$ and $\cosh x$ called the "Pythagorean-like identity" in lecture.

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- [a] Write that identity involving $\sinh x$ and $\cosh x$. You do NOT need to prove the identity.

$$\boxed{\cosh^2 x - \sinh^2 x = 1} \quad \textcircled{\frac{1}{2}}$$

- [b] Use the answer of [a] to find and prove an identity involving $\coth^2 x$ that resembles a Pythagorean identity from trigonometry.
NOTE: The proof shows how your answer came from the answer of [a].

$$\boxed{\frac{\cosh^2 x - \sinh^2 x}{\sinh^2 x} = \frac{1}{\sinh^2 x}} \quad \textcircled{\frac{1}{2}} \longrightarrow \boxed{\coth^2 x - 1 = \operatorname{csch}^2 x} \quad \textcircled{\frac{1}{2}}$$

- [c] Write the identity for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$ simultaneously. You do NOT need to prove the identity.

$$\boxed{\cosh 2x = \cosh^2 x + \sinh^2 x} \quad \textcircled{\frac{1}{2}}$$

- [d] Use the answers of [a] and [c] to find and prove an identity for $\cosh 2x$ that uses only $\sinh x$.

$$\boxed{\cosh^2 x = \sinh^2 x + 1} \quad \textcircled{\frac{1}{2}}$$

$$\boxed{\cosh 2x = \sinh^2 x + 1 + \sinh^2 x = 2\sinh^2 x + 1} \quad \textcircled{\frac{1}{2}}$$

- [e] If $\coth x = -\frac{5}{4}$, find $\operatorname{sech} x$ using identities. $\textcircled{\frac{1}{2}}$

You must explicitly show the use of the identities but you do NOT need to prove the identities.
Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.

$$\boxed{\tanh x = \frac{1}{\coth x} = -\frac{4}{5}} \quad \textcircled{\frac{1}{2}}$$

$$\boxed{\operatorname{sech}^2 x = 1 - \tanh^2 x}$$

$$= 1 - \frac{16}{25} \quad \textcircled{\frac{1}{2}}$$

$$= \frac{9}{25} \quad \textcircled{\frac{1}{2}}$$

$$\textcircled{\frac{1}{2}} \boxed{\operatorname{sech} x = \frac{3}{5}} \quad \left(\text{SINCE } \operatorname{sech} x = \frac{1}{\cosh x} \text{ AND } \cosh x > 0 \right) \quad \textcircled{\frac{1}{2}}$$